

# Method for determining the forces acting on a axle box designed for the installation of an adapter bearing

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## ARTICLE INFO

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**DOI:** 10.5937/engtoday2300010A

**UDC:** 621(497.11)

**ISSN:** 2812-9474

**Article history:** Received 8 June 2023; Revised 24 July 2023; Accepted 27 July 2023

## ABSTRACT

The installation of adapter bearings (cassette type) provides new opportunities to achieve longer maintenance intervals and to improve performance and safety. In the developed methodologies for calculating the forces acting on a axle box unit in European and state standards, dependencies are defined for axle boxes with short cylindrical rollers and conical or cassette type bearings where the transmission of the vertical load between the axle box and the bearings is on a cylindrical surface. In the contact area of a segment, in addition to the standard static, dynamic, centrifugal and inertial forces of the axle box, additional forces also act. Overloading occurs when the axle box has become detached from the bearing by lifting off of it. Under these conditions, the transmission of forces between the axle box and the bearing takes place not through the normal tight contact along a segment of 120°, but through the edge of the axle box shell. In this publication, a method is proposed for determining the forces in an axle box designed for the installation of an adapter bearing.

## KEYWORDS

Railway vehicles, Bogie, Axle box, Adapter bearing, Force.

## 1. INTRODUCTION

The axle boxes are designed to take the load from the wagon frame (bogie) and transmit it to the axle necks, ensure the rotation of the wheelsets with minimal resistance, limit the longitudinal and transverse displacement of the wheelsets. They must be strong, dense, provide good lubrication of the rubbing parts, light, convenient for inspection and repair.

In operation, there are various designs of railway axle boxes with installed spherical, cylindrical and cone-roller bearings [1,2,3]. The most common axle boxes designs in rolling stock are rolling bearing axle boxes with short cylindrical rollers. In figure 1 shows the axle box with jaw double-sided guidance, for spring suspension with cylindrical coil springs, for axle load - 225 kN and design speed of movement 120 km/h, production of TRANSWAGGON Burgas [4].

This design is relatively complex and operation depends on proper installation. Bearings must be manually lubricated and installed in a very clean environment to avoid contamination.

The installation of adapter bearings (compact tapered roller bearings) provides new opportunities to achieve longer maintenance intervals and improve performance and safety.

For many years, the trend in the rail industry has been to carry heavier loads at higher speeds. Today, most rail freight wagons in Europe are designed to run with an axle load of 25 t. SKF has developed a new axle box concept for the load bogie type Y25 [5], shown in figure 2, which is equipped with adapter bearings. This design offers opportunities to achieve lower life-cycle costs through longer maintenance intervals, simplified maintenance operations, and improvements in performance and safety.

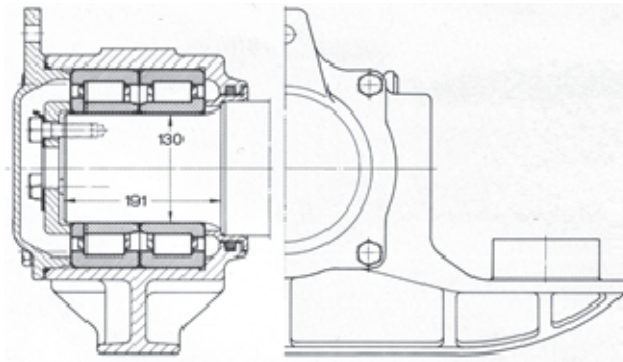


Figure 1: Axle box for freight bogie type Y25 manufactured by TRANSWAGON Burgas [4]

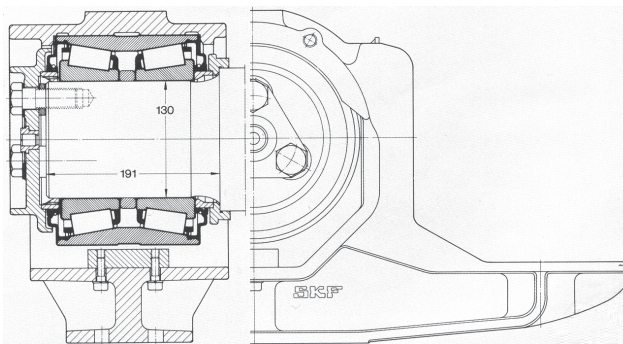


Figure 2: SKF axle box with adapter bearing [5]

The new SKF bearing design is based on the adapter bearing type CTBU 130 x 240 [6] (fig.2). This bearing assembly meets the requirements of the European standard EN 12 080 [7], focusing on the bearing material and quality requirements, as well as the EN 12 081 [8] standard on the lubrication specifications of the bearing grease. In addition, the CTBU 130 x 240 has passed the durability test requirements in accordance with the EN 12 082 standard [9]. SKF's test methods are much more difficult than actual operating conditions in service.

The main user benefits of the compact TBU bearing units [5] are:

- Shorter axle length for new bogie designs, reducing axle bending under the bearing and unsprung weight.
- Improved safety and performance through the use of polyamide cages instead of the traditional steel cages, resulting in less wear and reduced grease contamination.
- Avoidance of fretting corrosion in the inner ring contact zone, achieved by the use of a polymer spacer.
- Improved protection against contamination due to the incorporation of a newly designed low-friction contact seal that rides on the inner ring shoulder.
- Longer lubricant life, using new long-life grease.
- Simplified and easier mounting.

Such bearings are also produced by other companies, such as FAG [10], NTN [11], NSK [12], etc.

The axle box body is made of one or two parts (top and bottom connected by a bolt connection). The one-piece axle box body transmits the radial load through the top directly to the bearing assembly. Axial forces are transmitted in both directions through the fully integrated arms of the axle box housing.

No loads are transmitted through the lower part of the axle box body. The design has a large axial gap that serves to mount the bearing assembly in the newly designed arms of the upper part.

To prevent movement of the bearing out of the shoulders of the axle box during handling of the assembled wheelset in the workshop, a special locking wedge is applied. This fills the radial gap between the bearing unit outer ring and the lower part of the axlebox body and protects the complete assembly. The locking wedge is screwed axially onto the axle box body.

In the new Y25 axle box design, SKF completely redesigned the bushing body to achieve simplified installation and longer intervals between repairs, resulting in lower life cycle costs. The CTBU bearing design principle no longer requires a traditional full-bore axle box body with an integrated sealing system and front cover. This helps to save weight and simplify the construction of the axle box body by reducing the number of parts. The transmission of the vertical forces between the axle box body and the bearing is carried out by tight contact along a segment of 120°.

In the developed methods for calculating the forces acting on roller bearings [13-15] and bearing structures [16-18], dependencies are defined for calculating the forces loading bearings with short cylindrical rollers or conical bearings. In [1, 2, 3], methods for calculating a axle box unit are indicated, in which the transmission of the vertical load between the axle box body and the bearings is on a cylindrical surface. In the case of a contact area of a segment, additional loads appear which have not yet been determined.

In this publication, a method is proposed for determining the forces in the axle box body during contact between the bearing and the axle box body along a 120° segment.

Figure 3 shows a axle box for mounting an adapter bearing with "axle box body - bearing" contact through a 120° segment, for use in a Y25Lsad type freight bogie.

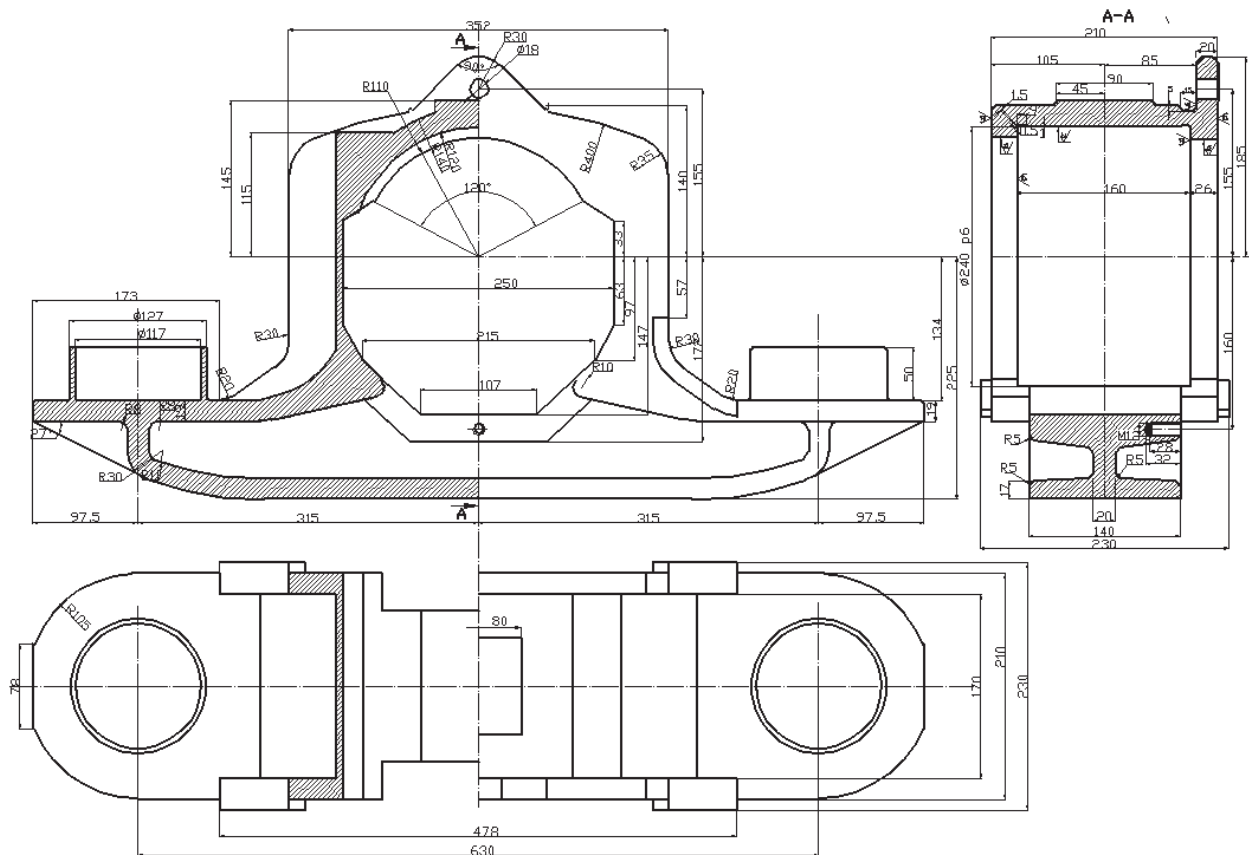


Figure 3: Axle box body for mounting adapter bearing

## 2. DETERMINATION OF FORCES ACTING ON A AXLE BOX BODY DESIGNED FOR MOUNTING AN ADAPTER BEARING

### 2.1. Load of a permanent nature

The load of permanent nature is the vertical static load under the action of tare and transported load.

#### a) Force loading a bogie

The force loading one bogie  $P_{br,t}$  is defined as half of the gross load of the wagon body  $P_{br,k}$ , i.e.

$$P_{br,t} = P_{br,k} / 2$$

or

$$P_{br,t} = P_{br,k} / 2 = (P_{br} - 2m_t) / 2$$

(1)

as:  $P_{br}$  – the gross load of the wagon,  $m_t$  – mass of one bogie.

The gross load of the wagon body -  $P_{br.k}$ , is determined by the following expression:

$$P_{br.k} = (T + Q - 2m_t) \cdot g \quad (2)$$

where:  $T$  - own mass of the wagon (tare);  $Q$  - maximum load capacity;  $g = 9.81 \text{ m/s}^2$  - ground acceleration;

b) Force acting on one axle box node

We assume that the axle box assemblies of one bogie take the same load, i.e. the load on the axle unit when the wagon is loaded -  $P_{bv}^{nat}$ , will be:

$$P_{bv}^{nat} = (P_{br.k} + 2m_t \cdot g - n \cdot g \cdot m_{neobr.}) / (2 \cdot m_2), \quad (3)$$

where:  $P_{br.k}$  - gross load of the basket;  $m_t$  - mass of one bogie;  $m_{neobr.}$  - mass of under spring parts of a wheelset;  $m_2$  - the number of axle box from one bogie;  $n$  - number of wheelsets in the wagon.

When the wagon is empty, the load on one axle box -  $P_{bv}^{pr}$  is determined by the equation:

$$P_{bv}^{pr} = (T \cdot g - n \cdot g \cdot m_{neobr.}) / (2 \cdot m_2). \quad (4)$$

## 2.2. Additional maximum vertical loads

The additional loads are the vertical load from the action of: dynamic load, centrifugal, from the action of the wind, braking and inertial forces.

Conditionally, we assume that the vertical loads act along the -  $z$  axis; longitudinal loads along the  $x$  axis, and transverse loads along the  $y$  axis.

### 2.2.1. With uniform movement of the wagon in a straight section of the track

During uniform movement of the wagon on a straight section of the road, additional maximum vertical loads arise, caused by the vibrations of the wagons during movement and the action of the wind force.

a) The vertical dynamic load is determined by the following formula:

$$P_{bv}^{din} = P_{bv}^{nat} \cdot k_{vd} \Rightarrow P_{bv}^{din} = P_{bv}^{nat} \cdot k_{vd} \quad (5)$$

where:  $P_{bv}^{nat}$  - load on the axle box when the wagon is loaded;  $k_{vd}$  - coefficient of vertical dynamics obtained experimentally or accepted value according to normative documents, determined in [1, 2, 3] (for freight wagons the value  $k_{vd} = 0.36$  is accepted).

b) Load from the action of the wind force

The load created by the wind acts on the side wall of the wagon, perpendicular to its longitudinal axis. The force of the wind -  $P_y$  is determined by the following formula:

$$P_y = F \cdot W \quad (6)$$

where:  $W$  - specific wind pressure (according to [1, 2, 3] for Bulgaria,  $W = 0,5 \text{ kN} / \text{m}^2$ );  $F$  - area of the side wall of the wagon;

$$F = H_k \cdot 2L$$

as:  $H_k$  - height of the wagon body with the load;  $2L$  - length of the wagon frame.

The wind force acting on one bogie -  $P_t^{vt}$ , is determined by the following formula:

$$P_t^{vt} = P_y \cdot h_v / (2 \cdot 2\ell_{pl}) \quad (7)$$

where:  $h_v$  - distance between the level of the slides and the geometric center of the wagon body;  $2\ell_{pl}$  - distance between the sliders.

Since the wind force -  $P_y$  acts in the geometric center of the wagon body, the distance  $h_v$  is equal to half the height of the basket, i.e. is determined by the formula:

$$h_v = H_k / 2$$

where:  $H_k$  - height of the wagon body;

\*the wind force acting on an axle box from a bogie.

We assume that the individual axle box of one bogie are subjected to the same load from the action of the wind force  $P_v$ , i.e. the wind force acting on one set -  $P_{bv}^{vt}$  will be:

$$P_{bv}^{vt} = P_t^{vt} / m_2 \tag{8}$$

The total load when moving in a straight section of the track will be determined by the equation:

$$P_{max}^z = P_{bv}^{nat} + P_{bv}^{din} + P_{bv}^{vt} \tag{9}$$

2.2.2. When the wagon is moving in a curve at maximum speed without slopping

a) Quasi-static loading

$$P_{bv}^{kvc} = P^c \cdot \frac{h_c}{2 \cdot b} = m_k \cdot \max a_H \cdot h_c / (2 \cdot b) \tag{10}$$

where:  $\max P_{bv}^{kvc}$  – maximum quasi-static force in a curve from the uncompensated traverse acceleration  $a_H$  (according to norms of various railways  $a_H = 1m / s$ );  $h_c$  – height of centrifugal force action (average according to expert assessment according to [1, 2, 3]);  $m$  – the mass of the wagon is distributed evenly between the four axles, according to the equation:

$$\Sigma M_0 = 0 \Rightarrow P^c \cdot h_c = 2 \cdot \max P_{bv}^{kvc} \cdot b.$$

The reactions in the rails are:  $\Delta Q_0 = \max P_{bv}^{kvc}$ .

b) Centrifugal force

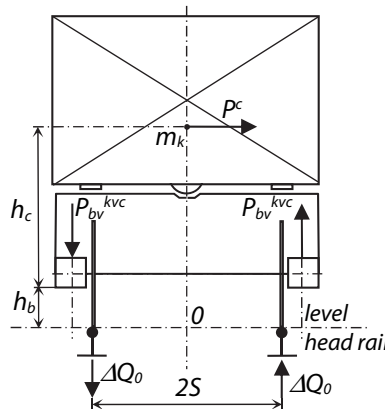


Figure 4: Forces in a curve

When the wagon passes through a curve, a centrifugal force  $P^c$  appears in its center of mass, which tends to tilt it outwards (Fig. 4), determined by:

$$P^c = P_{br.k} \left[ \frac{V^2}{g \cdot R} - \frac{h}{2 \cdot S} \right], kN \tag{11}$$

where:  $P_{br.k}$  – gross weight of the wagon body,  $V$  – the maximum speed of movement in a curve in  $m / s$ ;  $h$  – the maximum overhang of the outer rail in a curve;  $R$  – radius of the curve (according to [1, 2, 3]  $R = 250m$ );  $2S$  – distance between the rolling circles of the wheels of one wheel axle and with a gauge of 1435 mm -  $2S = 1.5m$ .

\*centrifugal force acting on a bogie

The centrifugal force acting on the bogie -  $P_t^c$  is determined by the following formula:

$$P_t^c = P^c \cdot h_c \cdot (2 \cdot 2 \ell_{pl}) \tag{12}$$

where:  $P^c$  – centrifugal force acting on the wagon;  $h_c$  – distance between the level of the sliders and the mass center of the wagon body.

\*centrifugal force acting on one axle box

We assume that the individual axle boxes of one bogie are subjected to the same load from the action of the centrifugal force  $P_t^c$ , i.e. the centrifugal force acting on one axle box -  $P_{bv}^c$  is:

$$P_{bv}^c = P_t^c / 4 \tag{13}$$

where:  $P_t^c$  – centrifugal force acting on the bogie.

Therefore, when driving in a curve without stopping, the total vertical load is:

$$P_{max}^z = P_{bv}^{nat} + P_{bv}^{din} + P_{bv}^{vt} + P_{vc}^c.$$

2.2.3. When moving the wagon in a curve at maximum speed and braking

When braking in the center of mass of the wagon body, an inertial force  $P^i$  acting horizontally and balanced by braking force between the wheel and the rails occur – (fig. 5).

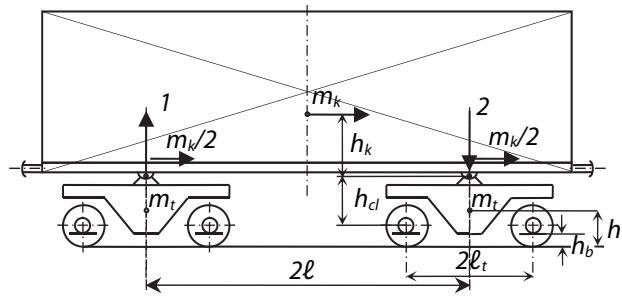


Figure 5: Inertia forces: 1 – force unloading rear bogie; 2 – force loading front bogie

Under these conditions, the braking force can be a maximum of 12% of the total mass of the wagon, i.e.

$$B_{sp} = P_{br} \cdot 0,12$$

where:  $B_{sp}$  – the braking force.

The longitudinal acceleration  $a_{sp}$  in the braking process will be:  $a_{sp} = B_{sp} / P_{br}$ ,  $m / s^2$ .

The total inertial force of the wagon is equal to the braking force:  $P^i = B_{sp}$ .

Then, from the total inertial force of the wagon, we will subtract the inertial force of the wheelsets  $P_{kol}^i$  (complete with the bushings and bearings).

We determine the inertial force of the wheelsets  $P_{kol}^i$  by the formula:

$$P_{kol}^i = n \cdot P_{kk} \cdot a_{sp}$$

where:  $n$  – number of wheelsets in a wagon;  $P_{kk}$  – approximately the mass of one wheelset with the two axle box and parts of the spring suspension.

The longitudinal inertial force of the sprung mass -  $R_{obr}^i$ , transmitted through the bushings will be:

- total

$$P_{obr}^i = B_{sp} - P_{kol}^i \tag{14}$$

- for one axle box

$$P_{bv}^i = P_{obr}^i / 8 \tag{15}$$

In "extreme braking" mode with acceleration -  $a_{sp} = -2m / s^2 \approx -0,2 \cdot g$

- the maximum additional braking force -  $max2P_{kol}^{sp}$ , which overloads the axle boxes from the first wheelset and unloads the axle boxes from the fourth wheelset is determined by the equation:

$$max2P_{kol}^{sp} = (m_k \cdot a_{sp} \cdot h_k) / 4l + (m_k / 2 \cdot a_{sp} \cdot h_{cl}) / 2l_t + [m_t \cdot s_{sp} \cdot (h_t - h_b)] / 2l \tag{16}$$

where:  $m_{br}$  – gross mass of the wagon;  $m_k$  – gross mass of the wagon body;  $m_t$  – mass of one bogie;  $2\ell$  – wagon base;  $2\ell_t$  – bogie base;  $h_k$  – height of the mass center of the wagon body above the central bearings;  $h_d$  – height of the central bearing above the support surface of the axle box above the rail head.

The total vertical force during the action of the brake in “extreme braking” mode is determined by the equation:

$$P_{max}^z = P_{bv}^{nat} + P_{bv}^{din} + P_{bv}^{vt} + P_{bv}^{kvc} + P_{bv}^i + P_{bv}^c + P_{bv}^{sp} \quad (17)$$

#### 2.2.4. Maximum longitudinal load

The maximum longitudinal load when moving the wagon in a straight line and in a curve at maximum operating speeds is the load with the brake engaged in extreme mode. Under these conditions, the braking force can be a maximum of 12% of the total mass of the wagon, i.e. the wheel will not slip on the rail.

$$P_{max}^x = P_{bv}^i \quad (18)$$

#### 2.2.5. Maximum transverse load

We determine the maximum transverse load of the axle box based on the international norms UIC-510-1 [19] according to which the maximum permissible value will be:

$$H = 0,2 \cdot 2Q_0$$

where:  $2Q_0$  – average static load on the wheelset.

The total flat (bearing) force is not evenly distributed between the two axle boxes due to deviations from the nominal characteristics. If we assume that the ratio between the forces of the two axle boxes is 60%/40%, the relevant transverse load  $P^y$  of the more strongly loaded axle box will be:

$$P_{max}^y = P_{bv}^y = 0,6 \cdot H \quad (19)$$

#### 2.2.6. Summary rating from maximum axle box loads

The maximum loads according to the preceding clauses (2.2.1), (2.2.2), (2.2.4) and (2.2.5) act jointly and the general condition of strength is that the axle box withstands this joint action

### 3. EXTRAORDINARY LOADS ON THE AXLE BOX

The forces with which the sockets are loaded can be determined experimentally by using instrumented wheelsets [20].

In addition to the standard static, dynamic, centrifugal and inertial forces of the axle box body for the installation of adapter bearings, additional forces also act.

Overloading occurs when the axle box has become detached from the bearing by lifting off of it. Under these conditions, the transmission of forces between the axle box and the bearing takes place not thorough the normal tight contact along a segment of  $120^\circ$ , but through the edge of the axle box shell, as:

- When a load from horizontal longitudinal acceleration  $< 5g$  occurs, it is not possible to “detach” the axle box shell (the  $120^\circ$  segment) from the bearing, if it was previously in a normal position;
- Under a load of horizontal longitudinal acceleration equal to  $5g$ , the return of the axle box shell to a normal position (if, for some reasons, it has become detached) will not be possible only for the axle boxes of the third wheelset (considered on the side of the impact).

#### 3.1. Extreme load in horizontal transverse direction

Extreme load in the horizontal transverse direction is regulated by the criterion of the UIC/ERRI code. UIC-510-1 [19] on the resistance of the track, by which the allowable load of the running gear is determined:

$$H + m_k \cdot \left( -y \right) \leq 0,85(10 + (2Q_0) / 3), kN \quad (20)$$

where:  $H$  – is the shoulder (bearing) force of the axle – which is unknown;  $m_k, y$  – the mass and transverse acceleration of the axle ( $y$  also – unknown);  $2Q_0$  – average static load of the axle on the rails, the so-called axle load.

When substituting  $2Q_0$  in the right-hand side of (20), for example for axle load  $2Q_0 = 25t = 245.25 \text{ kN}$ , we get:

$$H + m_k \cdot \left( -\ddot{y} \right) \leq 77.9875 \tag{21}$$

Due to the lack of data on the transverse acceleration  $(\ddot{y})$ , we will assume that  $(\ddot{y}) = 0$  because this case is the most unfavourable, i.e. the value of the shoulder (bearing) force  $H$  is the largest. Therefore, in the worst case scenario:

$$H < 77.9875 \tag{22}$$

If, due to deviations from the nominal characteristics, we allow unevenness in the distribution of the total force  $H$  between the two axle boxes in a ratio of 60%/40%, we obtain for the maximum value of the transverse force of one axle box  $P_{bv}^y$  in emergency situations:

$$P_{bv}^y = 0.6 \cdot H = 46.79 \text{ kN} \tag{23}$$

This transverse force ( $H_b = P_{bv}^y = 46.79 \text{ kN}$ ) acts between the wheelset and the frae through the axle box and the bearing (considered as intermediate bodies), whereby the direction of its transmission can be both "outward" and "inward".

### 3.2. Extreme load in horizontal longitudinal direction (from longitudinal impact)

We proceed from the code requirement. UIC-510-1 [19] to withstand one-time extreme loads on parts and joints in the undercarriage occurring at longitudinal acceleration  $(\ddot{x}) = 5g$  (under longitudinal impact conditions). With the

mass of the wheelset with the two axle boxes  $m_{kol}$  and acceleration in the longitudinal direction  $(\ddot{x}) = 5g$ , the inertial force in the same direction perceived by the two axle boxes  $2P_{bv}^{ix}$  will be:

$$2P_{bv}^{ix} = \ddot{x} \cdot m_k = 5g \cdot m_k \tag{24}$$

and for one axle box:  $P_{bv}^{ix} = 2.5g \cdot m_k$ .

### 3.3. Extreme vertical load on the axle box

#### A. Vertical additional load from the inertia force of the wagon body

The force  $\pm \Delta Q_{kosh}$  additionally overloading the first bogie and unloading the second bogie is obtained from the moment equation:

$$-\left( \ddot{x} \right) m_k \cdot h_k = \Delta Q_{kosh} \cdot 2\ell \tag{25}$$

where:  $m_k$  – gross mass of the wagon body;  $h_k$  – height of the mass center of the wagon body above the central bearing;  $2\ell$  - base of the wagon.

$$\Delta Q_{kosh} = -\left( \ddot{x} \right) \cdot m_k \cdot \frac{h_k}{2\ell} = \pm 5 \cdot g \cdot m_k \cdot \frac{h_k}{2\ell} \tag{26}$$

- For the whole bogie

$$\frac{\Delta Q_{kosh}}{4} = \Delta P_{kosh}^{iz} \tag{27}$$

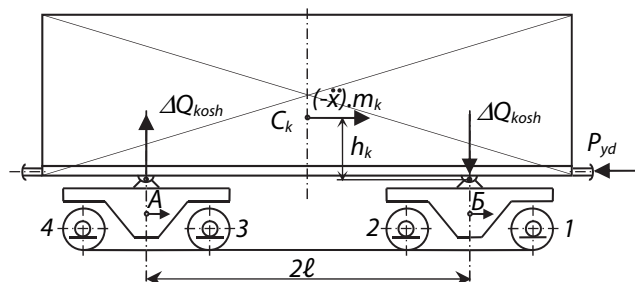


Figure 6: Scheme of additional vertical load from internal forces on the wagon body



- Additional loading of the axle boxes from the first bogie and unloading of the axle boxes from the second bogie due to the internal force of the wagon body.

B. Vertically additionally loaded by the internal force of the bogie

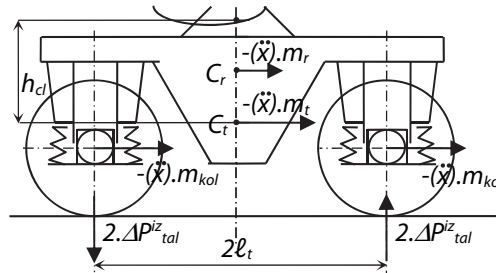


Figure 7: Scheme of additional vertical load from inertial forces on bogie

The force  $\pm\Delta P_{tal}^{iz}$ , which additionally loads the axle boxes from the second and fourth bogies and unloads the axle boxes from the first and third bogies, is obtained from the moment equality, causing a “rotation” of the bogie around the transverse axis passing through the central bearing, i.e.:

$$-(\ddot{x})m_t \cdot h_{cl} = 2\Delta P_{tal}^{iz} \cdot 2\ell_t \tag{28}$$

$$\Delta P_{tal}^{iz} = -(\ddot{x}) \cdot m_t \cdot \frac{h_{cl}}{4\ell_t} = \pm 5g \cdot m_t \cdot \frac{h_{cl}}{4\ell_t} \tag{29}$$

- additional loading of the axle boxes from the second and fourth wheelsets and unloading of the axle boxes from the first and third wheelsets, resulting in the bogie’s inertial force.

C. Total additional loading or unloading of the axle boxes on the various wheelsets

With the numbering of the wheelsets located relative to the side of the impact shown in Fig. 6, we have a total additional load or unloading  $\pm\Delta P_k^z$  on the individual wheelsets as follows:

$$\Delta P_{bv}^{iz} = \Delta P_{kosh}^{iz} \pm \Delta P_{tal}^{iz} \tag{30}$$

D. Total load on the axle boxes of the different wheelsets

$$P_{bv}^{iz} = P_{bv}^{nat} \pm \Delta P_{bv}^{iz} \tag{31}$$

**4. INVESTIGATION OF THE CONDITIONS FOR THE OCCURRENCE OF AXLE BOX DE-STOCK FROM THE BEARING**

4.1. Conditions for the axle box to detach from the bearing

Here we consider the extreme load case where the axle box has become detached from the bearing by lifting off it. Under these conditions, the transmission of forces between the axle box and the bearing takes place not through the normal tight contact along the 120° segment, but through the edge of the axle box shell.

The moment of detachment is considered when the gap between the segment of the axle box shell and the bearing is very small. Then the transmission of the forces takes place through the edge “A” (fig. 8).

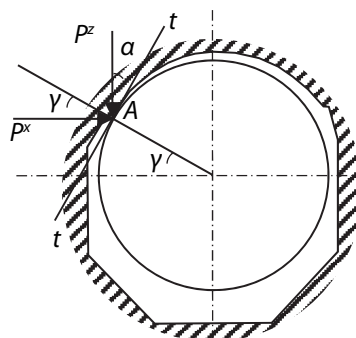


Figure 8: Diagram of “bearing-axle box shell” contact

The sum of the projections of the forces in the direction of the tangent t-t to the contact point is formed and the condition is sought under which the edge "A" of the axle box shell can slide in the "up" direction, i.e.

$$\sum p_{r_{sili}} - t \geq 0 \quad (32)$$

$$P^x \sin \gamma - P^z \cos \alpha \geq \mu \cdot P^x (\sin \alpha + \mu \cdot \cos \alpha)$$

or

$$P^x \cdot (\sin \gamma - \mu \cdot \cos \alpha) \geq P^z \cdot (\cos \gamma + \mu \cdot \sin \alpha)$$

or finally:

$$\frac{P^x}{P^z} \geq (\cos \gamma + \mu \cdot \sin \alpha) / (\sin \gamma - \mu \cdot \cos \alpha) \quad (33)$$

At  $\alpha = 30^\circ$  - (by design) and  $\mu = 0.3$  - (accepted), we get:

$$P^x / P^z \geq (\cos 30^\circ + 0.3 \cdot \sin 30^\circ) / (\sin 30^\circ - 0.3 \cdot \cos 30^\circ) \Rightarrow P^x / P^z \geq 4.23 \quad (34)$$

If it is assumed that, for some reason, the axle box shell has "detached" from the bearing, then the condition for it to take its normal state, i.e. to move in the "down" direction is:

$$P^z \cdot \cos \gamma - P^x \cdot \sin \alpha \geq \mu \cdot P^x \cdot \cos \alpha + \mu \cdot P^z \cdot \sin \gamma$$

or:

$$P^z \cdot (\cos \gamma - \mu \cdot \sin \gamma) \geq P^x \cdot (\sin \alpha + \mu \cdot \cos \alpha) \quad (35)$$

or finally:

$$P^x / P^z \leq (\cos \gamma - \mu \cdot \sin \gamma) / (\sin \alpha + \mu \cdot \cos \alpha) \quad (36)$$

at  $\alpha = 30^\circ$  and  $\mu = 0.3$  it is obtained:

$$\frac{P^x}{P^z} \leq \frac{\cos 30^\circ - 0.3 \cdot \sin 30^\circ}{\sin 30^\circ + 0.3 \cdot \cos 30^\circ} = \frac{0.866 - 0.3 \cdot 0.5}{0.5 + 0.3 \cdot 0.866} \Rightarrow \frac{P^x}{P^z} \leq 0.942 \quad (37)$$

#### 4.2. General conclusions and evaluation of the results regarding the conditions for the occurrence of the detachment of the axle box from the bearing

When substituting  $P_{bv}^{ix}$  and  $P_{bv}^{iz}$  in conditions (34) and (37) for the different bushings, the following conclusions are reached:

When a horizontal longitudinal acceleration load occurs  $x \leq 5g$ , it is not possible to "detach" the axle box shell (the  $120^\circ$  segment) from the bearing, if it was previously in a normal position.

2. Under load from horizontal longitudinal acceleration  $x = 5g$ , the return of the axle box shell to a normal position (if, for some reasons, it has come off) will not be possible only for the axle boxes of the third wheelset (considered on the side of the impact). But this applies only to the slice of time (moment) in which such acceleration acts.

3. Strength calculations of the axle box for the case of "detachment" from the bearing are not necessary, since this case does not actually exist.

When evaluating the results of the extraordinary loads on the axle box, the following should be done:

1. The extraordinary load in the horizontal transverse direction with the force  $P_{bv}^y$  on one axle box should be considered as an independent action.

2. The extraordinary load with the horizontal longitudinal force  $P_{bv}^{ix}$  and the vertical force  $P_{bv}^{iz}$ , to be treated under the joint action of these forces.

3. The equivalent stresses of the above-mentioned cases according to point 1 and point 2 are characterized by a single action; therefore they are allowed to reach the yield strength of the material  $\sigma_s$ .

### 5. DISTRIBUTION OF THE FORCES ACTING ON THE AXLE BOX

The force distribution on the axle box will initially be considered separately for the forces in the direction of the three coordinate axes: vertical - z, transverse - y and longitudinal - x, and in cases of combined (joint) action, the principle of superposition will be applied and the results will be summed up.

Regarding the values of the forces, the studies on their distribution will be specified for conventional values or for the results of extreme loads, and then they will be transferred to other loads in proportion to the actual values.

#### 5.1. Distribution of vertical forces

The forces transmitted by the axle box springs and detents will be evenly distributed over the areas corresponding to the springs, and the forces from the bearing ring on the contacting segment of 120° are characterized by an uneven distribution (fig.9).

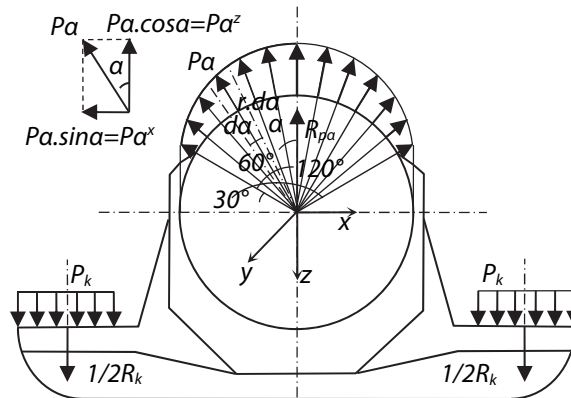


Figure 9: Diagram of distribution of vertical forces

To determine the law of this distribution, we proceed from published researches [13-15] on the normal forces on the rollers, according to which the greatest value  $P_{max}$  has the force of the uppermost roller (see Fig. 9) at  $\alpha = 0^\circ$ .

For the other rollers of the upper semicircle, located at arbitrary angles  $\alpha_i$  relative to the vertical, the normal forces  $P_{\alpha_i}$  are distributed according to the cosine law, i.e.

$$P_{\alpha_i} = P_{max} \cdot \cos \alpha_i \tag{38}$$

Since the dimensions, number and location of the rollers are not known for the adapter-bearings, here we will operate not with forces on the rollers, but with pressures. Based on (38), we will assume that the pressure  $p_\alpha$  on some elementary area  $ds$ , located at an arbitrary angle  $\alpha$  to the vertical z-axis, will obey the dependence:

$$p_\alpha = P_{max} \cdot \cos \alpha = p_0 \cdot \cos \alpha \tag{39}$$

where:  $p_{max} \equiv p_0$  – is the pressure at angle  $\alpha = 0^\circ$ .

But since the value  $p_{max} \equiv p_0$  is initially unknown, for its determination we will proceed from the set value of the vertical force -  $P^z$  on the axle box, which is balanced by the elementary forces formed by the vertical components of the pressures  $p_\alpha^z$ , determined by the relationship:

$$p_\alpha^z = p_\alpha \cdot \cos \alpha \tag{40}$$

or by substituting (39) into (40) we will have:

$$p_\alpha^z = p_0 \cdot \cos^2 \alpha \tag{41}$$

Given that the elementary areas -  $ds$  are of constant width -  $b$  (the width of the bearing ring),

$$ds = b \cdot r \cdot d\alpha \tag{42}$$

the elementary forces  $dp_\alpha^z$  for the strips of area  $ds$  will be:

$$dp_\alpha^z = p_\alpha^z \cdot ds \tag{43}$$

or

$$dp_\alpha^z = b \cdot r \cdot p_0 \cdot \cos^2 \alpha \cdot d\alpha \tag{44}$$

based on which we can form the integral:

$$P^z = 2 \int_0^{60^\circ} dp_\alpha^z = 2b \cdot r \cdot p_0 \cdot \int_0^{60^\circ} \cos^2 \alpha \cdot d\alpha \tag{45}$$

having the following decision:

$$P^z = 2b \cdot r \cdot p_0 \cdot [\pi / 6 + \sin 2\alpha / 4] \tag{46}$$

then for the pressure at angle  $\alpha = 0^\circ$  we get the equation:

$$p_0 = P^z / \{2b \cdot r \cdot [\pi / 6 + \sin 2\alpha / 4]\}$$

from which it is determined -  $p_0$

$$p_0 = \frac{P^z}{2b \cdot r \cdot \left[ \frac{\pi}{4} + \frac{\sin 120^\circ}{4} \right]}, \text{MPa}$$

Then equations (39) and (41) are specified as follows:

$$p_\alpha = p_0 \cdot \cos \alpha \tag{47}$$

$$p_\alpha^z = p_0 \cdot \cos^2 \alpha \tag{48}$$

The actively acting forces on the bushing, concentrated in the resultant  $R_k^z \equiv P^z$ , are those of the springs on the bushing wings, which forces are distributed uniformly at pressure  $p_k$  over the annular area  $S_k$ , determined by the overall dimensions of the spring set (the outer diameter  $D_{\text{on}}$  of the outer spring and the inner diameter  $d_{\text{in}}$  of the inner spring), i.e.

$$p_k = \frac{R_k}{S_k} = \frac{R_k}{2 \cdot \pi / 4 \cdot (D_{\text{on}}^2 - d_{\text{in}}^2)}, \text{MPa} \tag{49}$$

The reactive forces acting from the side of the bearing ring on the inner cylindrical surface of the axle box are concentrated in the resultant  $R_{pa}$ . The intensity of the distribution of these forces, respectively the normal pressure is determined by the cosine dependence (47). Their resultant  $R_{pa}$  is formed by the vertical components  $p_\alpha^z$ , and the horizontal components  $p_\alpha^x$  balance each other due to the symmetry of the distribution with respect to the vertical axis z.

### 5.2. Distribution of horizontal longitudinal forces

The force  $P^x$  in the longitudinal direction, acting externally on the axle box, is distributed over the surface of the end slide on one side, being balanced by the internally acting forces from the contact pressures in the bearing contact surface in the form of a 60° segment.

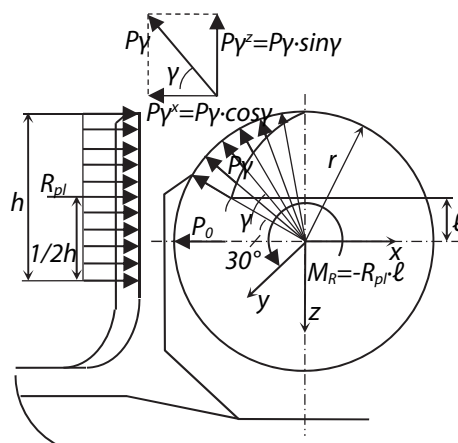


Figure 10: Diagram of distribution of horizontal longitudinal forces

Based on the same law for the distribution of forces on the rollers, as in the case of vertical loads [1-3] and proceeding here in basically the same way, we will assume that in one half (in this case - the left - fig. 10) of the contacting surface, the pressures  $P_y$ , depending on the angle  $\gamma$  relative to the horizontal, are distributed according to the cosine law, i.e.:

$$P_y = P_0 \cdot \cos \gamma \tag{50}$$

where  $P_0$  is a parameter corresponding to the value that the maximum pressure would have at an angle  $\gamma = 0^\circ$ .

Since the longitudinal force  $P^x$  acting on the box is balanced by the system of elementary forces  $dP^x$ , formed by the horizontal components  $P_y^x$  of the pressures  $P_y$  acting on the areas  $ds = b \cdot r \cdot d\gamma$ , and they (horizontal components  $P_y^x$ ) are determined by the equation:

$$P_y^x = P_y \cdot \cos \gamma \quad (51)$$

or

$$P_y^x = P_0 \cdot \cos^2 \gamma \quad (52)$$

can be written:

$$dP^x = b \cdot r \cdot P_0 \cdot \cos^2 \gamma \cdot d\gamma \quad (53)$$

and in integral form:

$$P^x = b \cdot r \cdot P_0 \cdot \int_{\gamma=30^\circ=\frac{\pi}{6}}^{\gamma=90^\circ=\frac{\pi}{2}} \cos^2 \gamma \cdot d\gamma \quad (54)$$

$$P^x = b \cdot r \cdot P_0 \left( \frac{\pi}{4} - \frac{\pi}{12} - \frac{\sin 60^\circ}{4} \right) \quad (55)$$

From where we determine the unknown parameter  $P_0$ :

$$P_0 = \frac{P^x}{b \cdot r \cdot \left( \frac{\pi}{6} - \frac{\sin 60^\circ}{4} \right)} \quad (56)$$

Then, in accordance with equations (50), (52), (56) and Fig. 10, the pressure formulas  $P_y$  and its components  $P_y^x$  and  $P_y^z$  take the form:

$$P_y = 5.617 \cdot \cos \gamma, \text{MPa} \quad (57)$$

$$P_y^x = 5.617 \cdot \cos^2 \gamma, \text{MPa} \quad (58)$$

$$P_y^z = 5.617 \cdot \sin \gamma \cdot \cos \gamma, \text{MPa} \quad (59)$$

When considering the scheme of forces on the axle box, it is important to bear in mind that the system of internal forces from the pressures  $P_y$ , acting as normal to the cylindrical surface, do not create a torque relative to the axis of wheelset. Therefore, in order not to create a torque and from the external forces transmitted through the front slide, it is necessary to the pivot support (i.e. to the axis of wheelset) to apply a balancing moment  $M_R$ , equal in absolute value and opposite in sign to the resultant torque  $R_{pl} \equiv P^x$  relative to the same axis. The absolute value of the moment  $M_R$  is determined by the product  $R_{pl} \cdot \ell$ , where the arm  $\ell$  is the distance between the axis of wheelset and the directrix of  $R_{pl}$ .

### 5.3. Distribution of horizontal transverse forces

During the extreme load in the horizontal transverse direction, the force  $P_{bv}^y$  acting on the more heavily loaded axle box is transmitted from the frame through the side sliders, exerting on them a normal pressure  $P_{pl}$  (in  $\text{N}/\text{m}^2$ ) with the resultant  $R_{pl}$  and through the springs (not shown in Fig. 11), which exert on the axle box wings a uniformly distributed tangential force with the resultant  $R_{kr}$  uniformly distributed on the surface with intensity  $\tau_{kr}$ ,  $\text{N}/\text{m}^2$ .

As a reaction from the side of the bearing on the board of the bushing (in section s-s fig.11), the pressure  $P_{lag}$  with the resultant  $R_{lag}$  and moment  $M_{lag}$  acts.

With a lateral clearance of  $\pm 5$  mm between the slides of the frame and the axle box and with the horizontal stiffness of the axle box springs  $c_{pr} = 4.92 \text{ kN}/\text{m}^2$  [3] the force from the extraordinary load  $P_{bv}^y$  will be distributed to  $R_{kr}$  and  $R_{pl}$  as follows:

- on the axle box wings  $R_{kr} = 5 \cdot c_{pr} i$
- on the axle box slides  $R_{pl} = P_{bv}^y - R_{kr}$ .

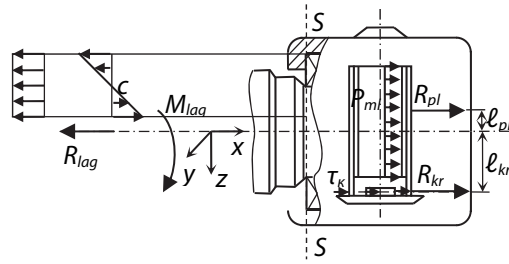


Figure 11: Diagram of distribution of horizontal transverse forces

In case the maximum load with lateral force -  $P_{max}^y$  acts, it will be distributed to  $R_{kr}$  and  $R_{pl}$  as follows:

- on the axle box wings  $R_{kr} = 5 \cdot c_{pr}$ ;
- on the axle box slides  $R_{pl} = P_{max}^y - R_{kr}$ .

The reactive result  $R_{lag}$  on the board of the axle box, transmitted from the bearing, is obviously the sum of the active forces acting on the side sliders -  $R_{pl}$  and the axle box wings  $R_{kr}$ , i.e.:

$$R_{lag} = R_{pl} + R_{kr}, \text{kN} \tag{60}$$

and the reactive moment under the same conditions balancing the active moment of  $R_{pl}$  and  $R_{kr}$  is:

$$M_{lag} = R_{kr} \cdot l_{kr} + R_{pl} \cdot l_{pl} \tag{61}$$

The reactive resultant  $R_{lag}$  forms the constant pressure  $P_{lag}$  at the surface of the board:

$$P_{lag} = \frac{R_{lag}}{S_{lag}}, \text{MPa} \tag{62}$$

where:  $S_{lag}$  is the area of the face surface of the flange of the section s-s which contacts the face surface of the bearing ring. The reactive moment  $M_{lag}$  forms the variable pressure  $P_{ml}$ , which is distributed over the front surface of the board according to a linear law along the z axis (fig. 11 - displayed laterally).

Following the prerequisites for displaying the bending formula [21], the variable pressure  $P_{ml}$  will be distributed according to the same formula, i.e.:

$$P_{ml} = M_{lag} \cdot \frac{z}{J_b}, \text{MPa} \tag{63}$$

where:  $J_b$  is the axial inertial moment on the square from the front surface of the board, contacting the bearing bracelet; z is the coordinate along the vertical axis, reported by the mass center of the same area. The maximum value (absolute value) of this stresses at the top and bottom point of the site will be:

$$\max P_{ml} = M_{lag} \cdot D_{vn} / 2J_b, \text{MPa} \tag{64}$$

where:  $D_{vn}$  is the outer diameter of the area of contact section (Fig. 12).

$$\max P_{ml} = M_{lag} \cdot \frac{z_g}{J_b}, \text{MPa} \tag{65}$$

$$\max P_{ml} = M_{lag} \cdot \frac{z_d}{J_b}, \text{MPa} \tag{66}$$

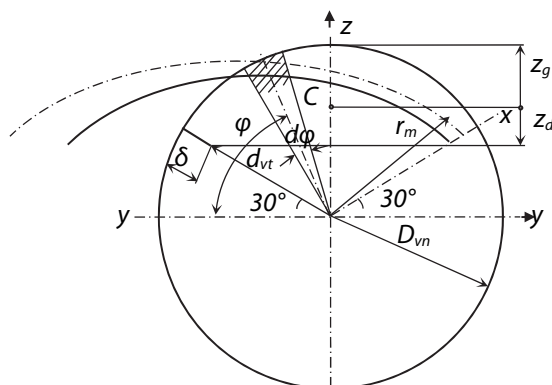


Figure 12: Scheme of the contact section

## 6. DETERMINATION OF LOADING AREAS

The maximum loads of the axle box body are distributed on the respective areas, namely: the longitudinal loads on the  $x$  axis - on the longitudinal axle box slides, the transverse (axial) on the axis  $y$  - on the transverse slides and the vertical loads on the  $z$  - on the wings of the axle box. The load from the  $P_{bv}^y$  will affect the area along the  $z$  axis.

A) Areas absorbing the load on the axis -  $x$

The face of the longitudinal axle box slide -  $F_{pl}^x$  with the accounting of the overlap of the bogie and axle box sliders with the  $a^{nadi}$ , loaded wagon, and the width of the  $b^{nadi}$  slide will be:

$$F_{pl}^x = a^{nadi} \cdot b^{nadi}, m^2 \quad (67)$$

B) Areas absorbing the axis of the axis -  $y$

The face of the transverse axle box slide -  $F_{pl}^y$  by taking into accounting of the overlap of the bogie and axle box sliders with a loaded wagon  $a^{nopr}$ , and the width of the  $b^{nopr}$  slide will be:

$$F_{pl}^y = a^{nopr} \cdot b^{nopr}, m^2 \quad (68)$$

C) Areas absorbing the axis -  $z$

The face of the area that is loaded along this axis is the one on which the coil springs are placed. At outer radius of the large spring  $r_{vn}$ :

$$F_{pr}^z = \pi \cdot r_{vn}^2, m^2 \quad (69)$$

## 7. APPLICATION OF A METHOD FOR DETERMINING THE FORCES ACTING ON AN AXLE BOX OF BOGIE TYPE Y25 Lsad

The parameters of Y25Lsad bogie freight wagon are:

$2Q_0 = 25 \text{ t} = 245.25 \text{ kN}$  – average static load on one wheelset;  $T = 25 \text{ t}$  - own mass of the wagon (tare);  $Q = 75 \text{ t}$  - maximum load capacity;  $m_m = 5.0968 \text{ t}$  - mass of one bogie;  $n = 4$  - number of wheelsets in the wagon;  $m_2 = 4$  - the number of axle boxes from one bogie;  $m_{neobr} = 1.325 \text{ t}$  - mass of unreinforced parts of a wheelset;  $2\ell = 15.800 \text{ m}$  - wagon base;  $2\ell_t = 1.8 \text{ m}$  - bogie base;  $h_c = 1.4 \text{ m}$  - distance between the level of the slides and the center of mass of the wagon body;  $h_k = 600 \text{ mm}$  – height of the mass center of the wagon body above the central bearings;  $h_{cl} = 584 \text{ mm}$  – height of the central bearing above the support surface of the axle box;  $h_t = 650 \text{ mm}$  - height of the center of mass of the bogie frame above the rail head;  $h_b = 316 \text{ mm}$  - height of the support surface of the axle box above the rail head.

### 7.1. Load of a permanent nature

a) force loading a bogie

$$P_{br,t} = 440.5 \text{ kN}$$

b) force acting on one axle box

$$P_{bv}^{nat} = 116.126 \text{ kN} - \text{when the wagon is loaded;}$$

$$P_{bv}^{pr} = 24.157 \text{ kN} - \text{when the wagon is empty.}$$

### 7.2. Additional maximum vertical loads

#### 7.2.1. Additional maximum vertical loads with uniform movement of the wagon in a straight section

a) vertical dynamic load

$$P_{bv}^{din} = 41.805 \text{ kN} - \text{of a axle box from one bogie.}$$

b) load from the action of the wind force

$$P_m^{vt} = 12.372 \text{ kN} - \text{on a bogie;}$$

$$P_{bv}^{vt} = 6.186 \text{ kN} - \text{of a axle box from one bogie.}$$

#### Total load when moving on a straight section of the railway track:

$$P_{max}^z = P_{bv}^{nat} + P_{bv}^{din} + P_{bv}^{vt} = 164,117 \text{ kN.}$$

## 7.2.2. Additional maximum vertical loads when moving the wagon in a curve

### 7.2.2.1. Additional maximum vertical loads when moving the wagon in a curve at maximum speed without braking

a) quasi-static load - maximum quasi-static force in a curve from the uncompensated transverse acceleration  $a_n = 1$  m/s (according to norms of different railways) –  $P_{bv}^{kvc} = 14.0156$  kN and  $\Delta Q_0 = \max P_{bv}^{kvc} = 14.0156$  kN,

b) centrifugal force

$P^c = 47.663$  kN, acting on the wagon;

$P_t^c = 19.626$  kN, acting on a single bogie;

$P_{bv}^c = 4.906$  kN, acting on one axle box.

### Total load when driving on a curve without braking

$$P_{max}^z = P_{bv}^{nat} + P_{bv}^{din} + P_{bv}^{vt} + P_{bv}^{ct} = 183.974 \text{ kN}$$

### 7.2.2.2. When moving the wagon in a curve at maximum speed and braking

a) inertial forces arising from braking

The braking force is a maximum of 12% of the total mass of the wagon, i.e.  $B_{sp} = 117.72$  kN.

The longitudinal acceleration  $a_{sp}$  in the braking process:  $a_{sp} = 1.1772$  m/s<sup>2</sup>.

The inertial force of the  $P_{kol}^i$  wheelsets (complete with axle boxes and bearings) is:  $P_{kol}^i = 6.239$  kN.

The longitudinal inertial force of the sprung mass –  $P_{obr}^i$ , transmitted through the axle boxes:

$P_{obr}^i = 111.48$  kN- total and

$P_{bv}^i = 13.935$  kN – for one axle box.

b) In "extreme braking" mode with acceleration –  $a_{sp} = -2$  m/s<sup>2</sup>  $\approx -0.2 \cdot g$

- maximum additional braking force -  $\max 2P_{kol}^{sp}$ , which overloads the axle boxes of the first wheelset and unloads the axle boxes of the fourth wheelset is:  $\max 2P_{kol}^{sp} = 34.473$  kN

- for axle box:  $\max P_{bv}^{sp} = 17.237$  kN.

### The total vertical force during the action and of the brake in "extreme braking" mode is:

$$P_{max}^z = P_{bv}^{nat} + P_{bv}^{din} + P_{bv}^{vt} + P_{bv}^{kvc} + P_{bv}^i + P_{bv}^c + P_{bv}^{sp} = 214.211 \text{ kN}$$

## 7.3. Maximum longitudinal load

The maximum longitudinal load when moving the wagon in a straight line and curve at maximum operational speeds is the load with the brake on in extreme mode:  $P_{max}^x = P_{bv}^i = 13.935$  kN.

## 7.4. Maximum transverse load

- total flat (bearing) force for one wheelset:  $H = 49.05$  kN;
- maximum transverse load  $P^y$  of the more heavily loaded axle box will be:  $P_{max}^y = P_{bv}^y = 29.43$  kN.

## 7.5. Extreme load in horizontal longitudinal direction (from longitudinal impact) with longitudinal acceleration ( $x$ ) = $5g$

- the inertial force in the same direction perceived by the axle box:  $P_{bv}^{ix} = 32.496$  kN.

## 7.6. Extreme vertical load on the axle box

- additional loading of the axle boxes from the first bogie and unloading of the axle boxes from the second bogie due to the inertial force of the wagon body:  $\Delta P_{kosh}^{iz} = \pm 41.910$  kN;
- additional loading of the axle boxes from the second and fourth wheelsets and unloading of the axle boxes from the first and third wheelsets, as a result the inertial force of the bogie:  $\Delta P_{tal}^{iz} = \pm 23.844$  kN;
- total additional loading or unloading of  $\Delta P_{bv}^{iz}$  axle boxes on the various wheelset:



For one axle box of:

1-st wheelset: 18.066 kN;

2-nd wheelset: 65.754 kN;

3-rd wheelset: 65.754 kN;

4-th wheelset: 18.066 kN.

- total load on the axle boxes of the different wheelset:

For one axle box of:

1-st wheelset: 134.192 kN;

2-nd wheelset: 1181.880 kN;

3-rd wheelset: 50.372 kN;

4-th wheelset: 98.060 kN.

## 8. CONCLUSION

The paper examines the main design solutions for wagon axle boxes. It is focused on the features related to the use of adapter bearings.

A method has been developed for determining the forces loading axle box body, designed for the installation of an adapter bearing. Dependencies are indicated for determining the forces of constant nature acting on the wagon body, the bogie and the axle box of the wagon. The forces loading the axle box caused by dynamic, centrifugal, inertial, braking forces and the action of the wind are considered. Special attention is paid to extreme loads on the axle box body in conditions of detachment of the axle box from the bearing.

Dependencies are defined for determining the forces that additionally load the individual elements of the axle box.

The presented method is used in the calculation of the slats in the axle box for a freight wagon with bogie type Y25 Lsd for an axle load of 25 t.

Based on calculations, the maximum load on the axle box was determined when the wagon was moving on a straight and curved section of the railway track with and without braking, as well as under extreme loads from the inertial force, from impact and the case of the axle box coming off the bearing.

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